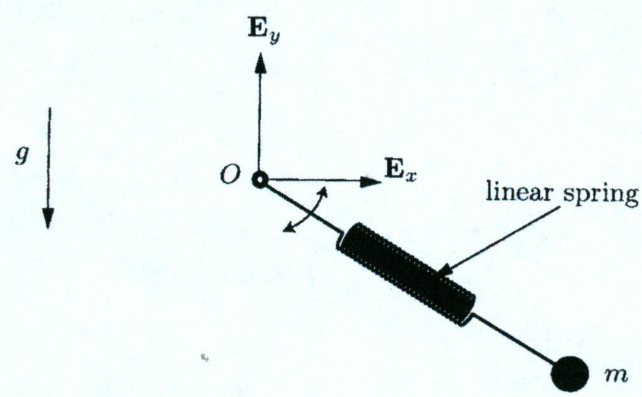
Kira Neuman

Mechanical Engineering 230—Dynamics
Midterm Exam
Rico A.R. Picone
28 April 2014

Directions: 50 minutes, calculator, open notes, closed book. Use your own paper, work neatly, and clearly mark your answers. Partial credit may be given. Do no work on this sheet.

Problem 1 (30 points). The figure shows a particle of mass m attached to a fixed point O by a linearly elastic spring. The spring has stiffness K and unstretched length L. The motion of the particle is in the $\mathbf{E}_x - \mathbf{E}_y$ plane. A vertical gravitational force acts on the particle in the *negative* $\mathbf{E}_y$-direction.



(a) Starting from the standard representations for the position vector

$$\mathbf{r} = x\mathbf{E}_x + y\mathbf{E}_y = r\mathbf{e}_r \tag{1}$$

for the particle, derive expressions for the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ vectors in the given Cartesian *and* the standard polar coordinate bases.
Hint (no need to derive): $\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta$ and $\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$.

(b) Draw a free-body diagram of the particle and write what is known about each force acting on the particle.

(c) Show that the scalar differential equations governing the motion of the particle are, in polar coordinates,

$$-K(r-L) - mg\sin\theta = m\left(\ddot{r} - r\dot{\theta}^2\right) \tag{2a}$$

$$-mg\cos\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right). \tag{2b}$$

(d) What law of motion did you use to find (2a) and (2b)?

(e) What must r and θ be if the particle is at rest?

Problem 2 (20 points). Give short descriptions (one-to-two sentences) of the following.

(a) particles

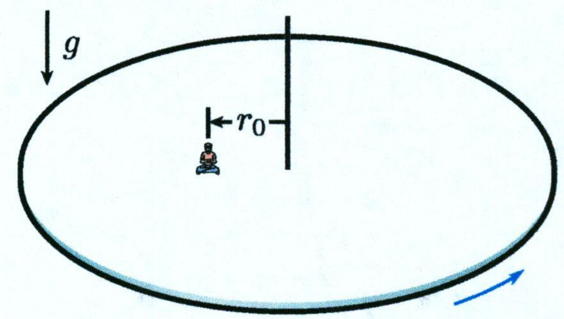
(b) force

(c) (coordinate) bases

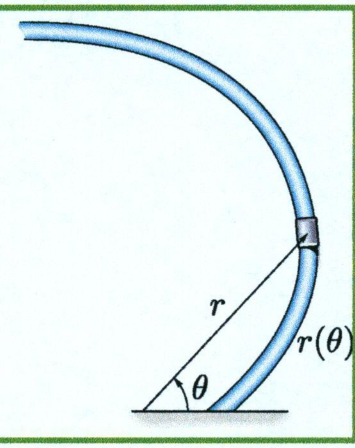
(d) the Sennet-Frenet basis

(e) friction forces

Problem 3 (30 points). The figure shows a boy of mass m sitting on a disk, a distance r_0 from its center. If the disk, starting from rest, begins spinning such that the boy's speed v increases at a constant rate $\dot{v} = a_0$, how long until the boy slips? The coefficient of static friction between the boy and the disc is μ_s. Write your answer in terms of m, r_0, a_0, μ_s, and the gravitational constant g. Does a_0 have a maximum above which the boy slips immediately?
If not, explain; if so, what is it?



Problem 4 (20 points). The figure shows a collar with mass, sliding on a rail with known shape described by $r(\theta)$. Explain in detail *how* you would find the equation of motion of the particle. Describe *why* you would take each step. Do not actually perform the work, just describe what you *would* do.



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Problem 1

a) $\bar{r} = x\bar{E}_x + y\bar{E}_y = r\bar{e}_r \rightarrow \boxed{\bar{r} = \cancel{(r\sin\theta)\bar{E}_x + (r\cos\theta)\bar{E}_y} = r\bar{e}_r}$ ✓

$\dot{\bar{r}} = \bar{v}$

$\bar{v} = \dot{x}\bar{E}_x + \dot{y}\bar{E}_y = \dot{r}\bar{e}_r + r\dot{\theta}\bar{e}_\theta \rightarrow \boxed{\bar{v} = \cancel{r\cos\theta\bar{E}_x - (r\sin\theta)\bar{E}_y} = \dot{r}\bar{e}_r + r\dot{\theta}\bar{e}_\theta}$ ✓

$\cos\theta = \frac{y}{r} \rightarrow y = r\cos\theta$

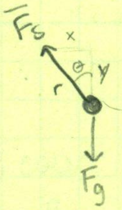
$\sin\theta = \frac{x}{r} \rightarrow x = r\sin\theta$

$\dot{\bar{v}} = \bar{a}$

$\bar{a} = \ddot{x}\bar{E}_x + \ddot{y}\bar{E}_y = (\ddot{r} - r\dot{\theta}^2)\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{e}_\theta$

$\boxed{\bar{a} = \cancel{(-r\sin\theta)\bar{E}_x - (r\cos\theta)\bar{E}_y} = (\ddot{r} - r\dot{\theta}^2)\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{e}_\theta}$ ✓

these are correct but not necessary. You'd need to apply chain rule for a, x in Cartesian ✓

b) 

$\bar{F}_g = -mg\bar{E}_y = ma_y$ ✓

$\bar{F}_s = -k(\underbrace{\|r - r_A\| - L}_{(r-L)})\bar{e}_r$

$\bar{F} = m\bar{a} \rightarrow \bar{F}_g + \bar{F}_s = m\bar{a}$

$-mg\bar{E}_y - k(r-L)\bar{e}_r = m((-r\sin\theta)\bar{E}_x - (r\cos\theta)\bar{E}_y) = m((\ddot{r} - r\dot{\theta}^2)\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{e}_\theta)$

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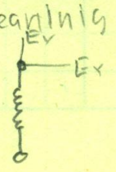
d) Newton's second law -5

e) if the particle is at rest the spring will be vertical meaning $r = -k(\|0 - r_A\| - L))\bar{E}_y$ ~ Should also be a function of mg



$\theta = 270°$ ✓ measure from $\bar{E}_x$

-5

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Problem 2

a) particles: simplification of a body of interest so that the effects of its size and shape can be neglected; a point in space +4 internal structure can be neglected

b) force: $\bar{F} = m\bar{a}$, a force acts on a body and is represented by a magnitude and direction +4

c) coordinate bases: the chosen way to describe the "setting" of a problem relative to an origin. Basis vectors are orthogonal. Example? used to describe position +3

d) Sennet-Frenet basis:

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e) Friction forces: forces caused by contact between two surfaces/materials, opose motion +3
proportional to normal force

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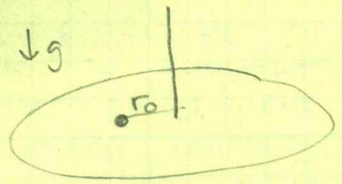
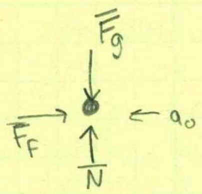
Problem 3:



mass m, starts at rest $v_0 = 0$

$\dot{v} = a_0 \quad \mu_s$

How long until the boy slips? The boy will slip when the force cause by rotation overcomes the force of friction



$\|\bar{F}_f\| = \mu_s \|\bar{N}\|$ +3

$\bar{F}_g = -mg \rightarrow \bar{N} = mg \qquad F_f = -ma_0$

$\|F_f\| = \mu_s mg$

if not slipping $r_0 = r_0 \quad \dot{r} = \ddot{r} = 0$

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a_0

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Problem 4:

To Find the equation of motion:
select a base: polar is a good choice for this problem since we have r and θ
Kinematics: write down equations for $\bar{r}, \bar{v}, \bar{a}$ using known values/equations and derivatives/integrals and any unknowns necessary

Forces: draw a free body diagram and write the force balances. Relate these to $\bar{F}=m\bar{a}$

Combining force balance w/ $\bar{F}=m\bar{a}$ allows us to solve for an unknown that appears in one of these. Since this problem is looking for equation of motion, there was probably an unknown in the equation for $\bar{a}$ from kinematics

Analyze + Solve: rearrange equations to find the desired output (equation of motion) and plug in any remaining variable if applicible.

origin of coordinate system?

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